

A Note on Disk Drag Dynamics

Neil J. Gunther

Performance Dynamics Company, Castro Valley, CA 94552
njgunther@perfdynamics.com

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Abstract

The electrical power consumed by typical magnetic hard disk drives (HDD) not only increases linearly with the number of spindles but, more significantly, it increases as very fast power-laws of speed (RPM) and diameter. Since the theoretical basis for this relationship is neither well-known nor readily accessible in the literature, we show how these exponents arise from aerodynamic disk drag and discuss their import for green storage capacity planning.

1 Introduction

The semi-empirical relationship [1, 2, 3]

$$\text{Power} \propto \text{Platters} \times \text{RPM}^{2.8} \times \text{Diameter}^{4.6}, \quad (1)$$

has been used to motivate “greener” magnetic HDD designs [4]. Of the three variables that can be used to match a given power constraint, diameter and RPM (*revolutions per minute*) have the greatest impact due to the high degree of their respective positive exponents (Fig. 1). It is noteworthy that (1) is not a function of capacity (e.g., GB); the typical metric used for storage capacity planning.

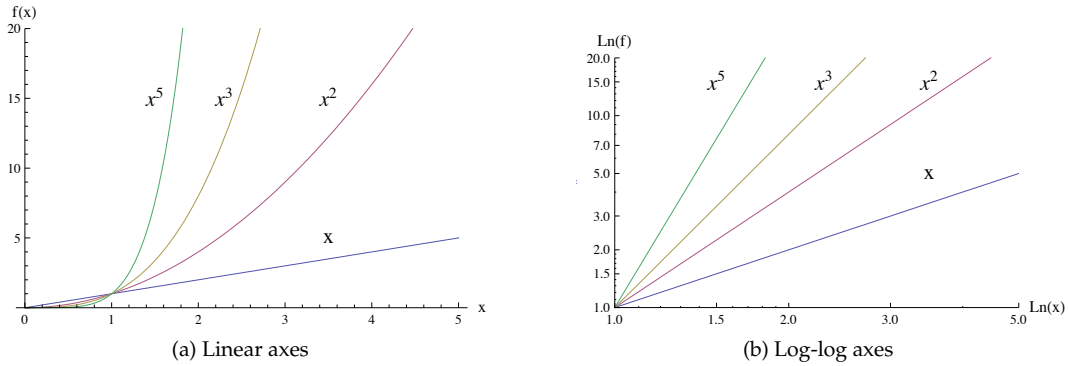


Figure 1: A fast function family: $f(x) = x^p$ with powers $p = 1, 2, 3, 5$.

The HDDs used in laptops are designed to operate in the energy-limited environment of a battery-powered system. Consequently, those disks tend to be smaller, slower, and have fewer platters. Server HDDs, on the other hand, tend to follow the converse trend. Since the RPM can be varied dynamically (DRPM) [4], disk drive manufacturers now offer this capability in some models [5].

2 Calculating Power

For the purposes of comparing the power consumption of two HDD models (say, a and b)¹ it is more convenient to express (1) in the *ratio form*

$$\frac{\mathcal{P}_b}{\mathcal{P}_a} = \left(\frac{N_b}{N_a}\right) \left(\frac{\Omega_b}{\Omega_a}\right)^{2.8} \left(\frac{D_b}{D_a}\right)^{4.6} \quad (2)$$

which avoids the necessity of otherwise determining a constant of proportionality.

In this simpler notation, \mathcal{P} is the power consumed (Watts), N is the number of platters per spindle, Ω is the angular speed (RPM), and D is the platter diameter (not the external form factor) usually expressed in *inches* in U.S. vendor data sheets. Another convenience of using (2) is that the ratio of two HDD parameters that are common (e.g., the same diameter) simplifies to unit value.

Example 1 (RPM Variation). *The parameters in the following table are for HDDs with a single platter $N_a = N_b = 1$ and the same diameter $D_a = D_b = 2.6$ inches [4].*

Single platter 2.6 inch HDD				
\mathcal{P}_a	\mathcal{P}_b	Ω_a	Ω_b	\mathcal{P}_b^*
0.91	1.13	15,098	16,263	1.121
2	35.55	19,972	55,819	35.550
35.55	499.73	55,819	143,470	499.782

Hence, only the ratio of the angular speeds contributes in (2) to the estimated power \mathcal{P}_b^* in the last column.

Because the exponents in (1) are relatively large, the functions of RPM and diameter are highly nonlinear and that means:

- Potentially large energy savings can be achieved even within a limited selection of HDD models, especially when taken in aggregate across SANs, NAS, JBODs, or RAID storage configurations.
- The key disk parameters are available from vendor data sheets, although some caveats still apply [6].

Example 2 (Diameter Variation). *The data used in this example are taken from Tables 3.3–3.5 in 1. The effect of reducing HDD diameter (d) is shown in Fig. 2. At highest RPM, both the $d = 2.6$ inch and $d = 2.1$ inch HDDs are located higher on the power hill than the $d = 1.6$ inch HDD.*

3 Theoretical Justification

We now turn to providing a theoretical justification for the semi-empirical relationship in (1). As a starting point, we assume:

1. The empirical exponents can be associated with integers (rounded up)
2. Planar rotation of the platter imposes axial symmetry
3. Rotational friction is present since the platter is not *in vacuo*

A thin rotating platter implies that inertial linear relationships, like the kinetic energy alone, cannot produce (1). Rather, rotational quantities, like moments of inertia, are more important.

Rotation also implies that there is aerodynamic friction due to the platter spinning. This is also assumed to be greater than friction in the spindle bearings. Since the HDD platter resides in a stationary housing, we assume there is no translational drag proportional the cross-sectional area of the platter; as there would be for a fan-blade or propeller pulling air.

¹Such as the kind of calculations required for data center and storage capacity planning.

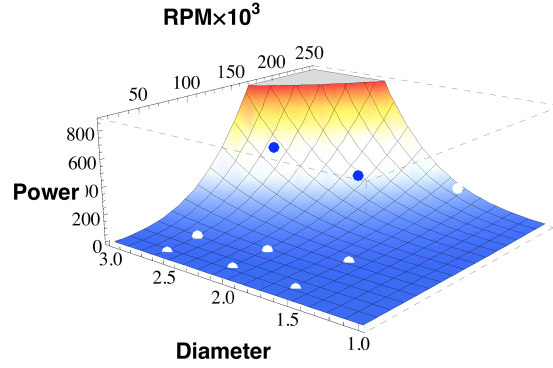


Figure 2: Specific HDD configurations (*dots*) plotted on the power surface defined by (1). The design trend is to move toward the bottom corner of the HDD power surface.

3.1 Pressure

The Bernoulli equation tells us that the pressure at some point in a fluid (e.g., air with density ρ) is given by:

$$P_p = p + \rho gh + \frac{1}{2}\rho v^2.$$

For the spinning disk, we are only interested in the dynamic contribution of the third term. The external and hydrostatic contributions can be ignored.

Since pressure is force per unit area (F/A), the aerodynamic frictional force will be

$$F_p = \frac{1}{2}\rho v^2 C_d A, \quad (3)$$

where C_d , the *drag coefficient* is an additional fudge factor that covers a multitude of sins as to how the fluid drag actually occurs².

The drag force in (3) is to be understood as being tangential, rather than centripetal. The velocity v at any point on the platter surface has magnitude relative to the air in the tangential direction of the disk spin. Similarly, the area A is on the surface of the platter (not its edge) where the air literally drags. The roughness of the platter surface is captured in C_d .

3.2 Power

In its mechanical form, power is the rate of doing work (W) or expending energy.

$$\mathcal{P} = \frac{dW}{dt}. \quad (4)$$

For an HDD, however, it's probably more convenient to measure the energy loss due to aerodynamic friction in terms of the current drawn (I) at voltage (V):

$$\mathcal{P} = IV = \frac{dq}{dt} V. \quad (5)$$

If we count the charges q as electrons, then dqV is measured in electron-volts (eV), which is a measure of energy³. Hence, (4) and (5) are dimensionally equivalent.

²Factors such as: shape, roughness, viscosity, compressibility, boundary-layer separation, etc.

³KeV in an analog TV.

Moreover, the work performed in (4) is by virtue of the friction force in (3) acting along an elemental (tangential) arc length ds , i.e., $dW = F_\rho ds$. Hence, the power

$$\mathcal{P} = F_\rho \frac{ds}{dt} = F_\rho v, \quad (6)$$

can be expressed directly in terms of the drag force and the tangential velocity.

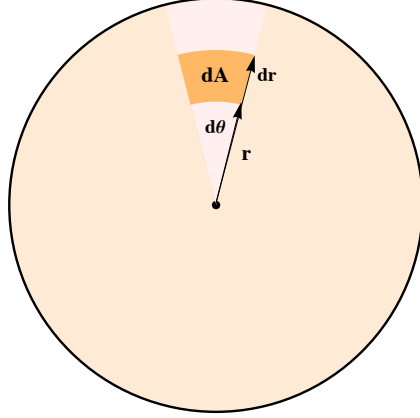


Figure 3: Differential area element dA defined on a disk platter

However, each elemental area (dA) on the platter moves with a different tangential speed and therefore experiences a different force. Thus, we have to integrate all possible patches over the whole platter. Applying (3), the power integral can be written as:

$$\mathcal{P} \propto \int \rho v^3 dA, \quad (7)$$

ignoring any proportionality constant.

From Fig. 3 the sectorial area on the platter can be written as the product of the arc delta (ds) and radial delta (dr):

$$dA = ds \cdot dr = (r d\theta) dr. \quad (8)$$

Similarly, the tangential velocity vector \mathbf{v} is related to the (axial) angular velocity $\mathbf{\Omega}$ by $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$ but, since all these vectors are orthogonal on the disk we can use the scalar form

$$v = |\mathbf{\Omega}| r. \quad (9)$$

Substituting (8) and (9) into (7) produces the two-dimensional integral

$$\mathcal{P} \propto \rho \int_0^R dr \int_0^{2\pi} r d\theta r^3 \Omega^3 \equiv \rho \Omega^3 \int_0^R dr r^4 \int_0^{2\pi} d\theta,$$

which yields

$$\mathcal{P} \propto \frac{2\pi}{5} \rho \Omega^3 R^5. \quad (10)$$

With $D = 2R$, (10) is identical to (1) up to constants of proportionality.

4 Summary

The theoretical contributions to (1) can be summarize in the following simple steps:

1. Factor the power \mathcal{P} as $\Omega^3 R^3 \times R^2$
2. R^2 comes from the total area of the platter
3. ΩR is the angular speed v of air at any point on the spinning platter
4. Why the cube: $v^3 \equiv \Omega^3 R^3$?
5. A factor v^2 comes from the Bernoulli pressure (F/A) in (3)
6. Another v factor comes from the definition of power $\mathcal{P} = v \times F/A$ in (6)

A factor of 2 should also be included since the drag will occur on both the upper and lower side of the platter, but we are not keeping track of constants here. There could also be inter-platter turbulence but the Reynold's number is likely already substantial due to head movement.

It has also been shown recently that electrical power consumption is proportional to the third power of the logical block number [7]. This is likely related to Ω^3 via (9).

The preceding discussion concerning HDD aerodynamic power consumption will become moot as SSDs become increasingly cost-effective and reliable [8, 9, 10].

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